Hints for 13.2 from yesterday:

25. $$\lim_{h\to 0}\frac{(2+h)^2-4}{h} = \frac{(2+h)(2+h)-4}{h}$$ multiply & simplify

29. $$\lim_{x\to -4}\frac{\left(\frac{x\cdot 1}{x\cdot 4}+\frac{1\cdot 4}{x\cdot 4}\right)}{4+x} = \frac{\frac{x}{4x}+\frac{4}{4x}}{4+x} = \frac{\frac{x+4}{4x}}{\frac{4+x}{1}}$$

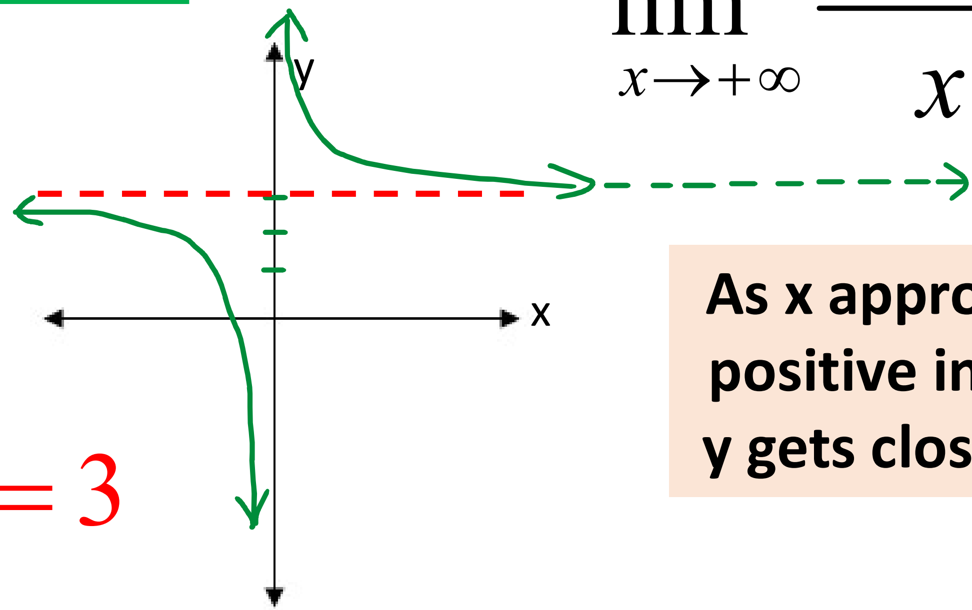
$$= \frac{\cancel{x+4}^{\,1}}{4x}\cdot\frac{1}{\cancel{4+x}_{\,1}} = \frac{1}{4x} = \boxed{-\frac{1}{16}}$$

-4

Notes: 13.4 Limits at Infinity

From yesterday:

$$\lim_{x \to +\infty} \frac{1+3x}{x} = 3$$



As x approaches positive infinity, y gets close to 3.

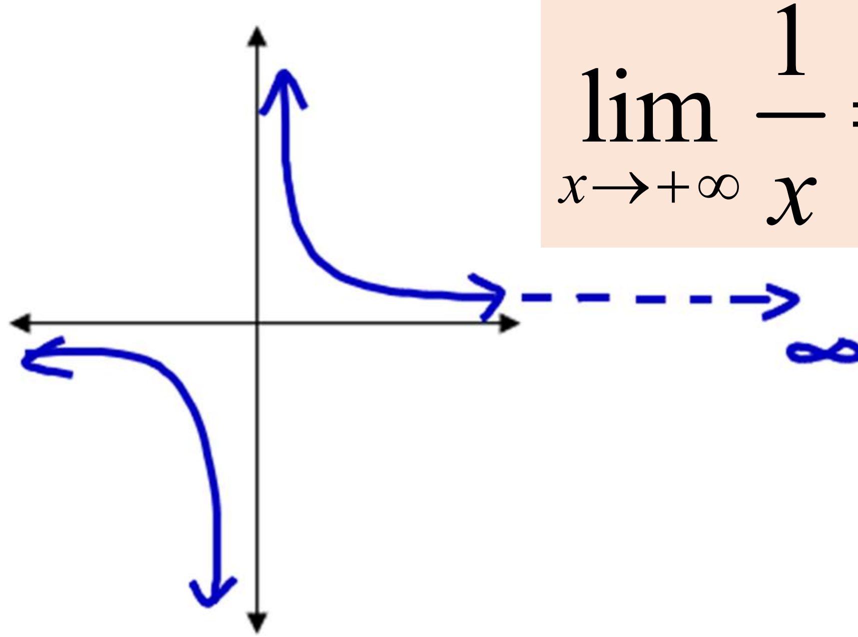
$$\lim_{x \to -\infty} \frac{1+3x}{x} = 3$$

As x approaches negative infinity, y gets close to 3.

COMPARE:

graph of

$$y = \frac{1}{x}$$



$$\lim_{x \to +\infty} \frac{1}{x} = 0$$

Consider some positive values substituted for x:

$$\frac{1}{0}, \quad \frac{1}{1/10}, \quad \frac{1}{1/7}, \quad \frac{1}{1/2}, \quad \frac{1}{1}, \quad \frac{1}{2}, \quad \frac{1}{3}, \quad \frac{1}{4} \quad \cdots$$

Undefined (asymptote)

=10

=7

=2

COMPARE:

The terms are getting closer to zero ↓

Given sequence: $1, \ \frac{1}{2}, \ \frac{1}{3}, \ \frac{1}{4}, \ \frac{1}{5}, \ \dots, \frac{1}{n}$

Therefore: $\boxed{\lim_{n\to\infty} \frac{1}{n} = 0}$

NOTES: examples

Evaluate:

a) $\lim_{x \to -\infty} \dfrac{12}{x}$

$= \boxed{0}$

b) $\lim_{x \to \infty} \dfrac{3}{x^2}$

$= \boxed{0}$

c) $\lim_{x \to \infty} \dfrac{x^2}{3}$

$= \infty$

$= \boxed{\text{dne}}$

limit does not exist

NOTES: continued...

d) $$\lim_{x\to\infty}\frac{12x+1}{5x-9} = \lim_{x\to\infty}\left(\frac{\frac{12\cancel{x}}{\cancel{x}} + \frac{1}{x}}{\frac{5\cancel{x}}{\cancel{x}} - \frac{9}{x}}\right)$$

divide

Simplify

$$= \lim_{x\to\infty}\left(\frac{12+\frac{1}{x}}{5-\frac{9}{x}}\right)$$

$$= \frac{12+0}{5-0} = \boxed{\frac{12}{5}}$$

apply limit

***Divide all terms by the <u>highest</u> power of x.**

NOTES: continued...

e) $\lim\limits_{x\to\infty} \dfrac{3x^2-5}{2-x^3}$ divide $= \lim\limits_{x\to\infty}\left(\dfrac{\dfrac{3\cancel{x^2}}{\cancel{x^3}} - \dfrac{5}{x^3}}{\dfrac{2}{x^3} - \dfrac{x^3}{x^3}}\right)$

simplify $= \lim\limits_{x\to\infty}\left(\dfrac{\dfrac{3}{x} - \dfrac{5}{x^3}}{\dfrac{2}{x^3} - 1}\right)$

***Divide all terms by the <u>highest</u> power of x.**

now apply limit $= \dfrac{0-0}{0-1}$

$= \dfrac{0}{-1} = \boxed{0}$

NOTES: continued...

f) $$\lim_{x\to\infty}\frac{4x^2}{5-2x^2}-\left(\frac{3}{x}+7\right)$$

as is   as is

$$+\lim_{x\to\infty}\left(\frac{-3}{x}+7\right)$$

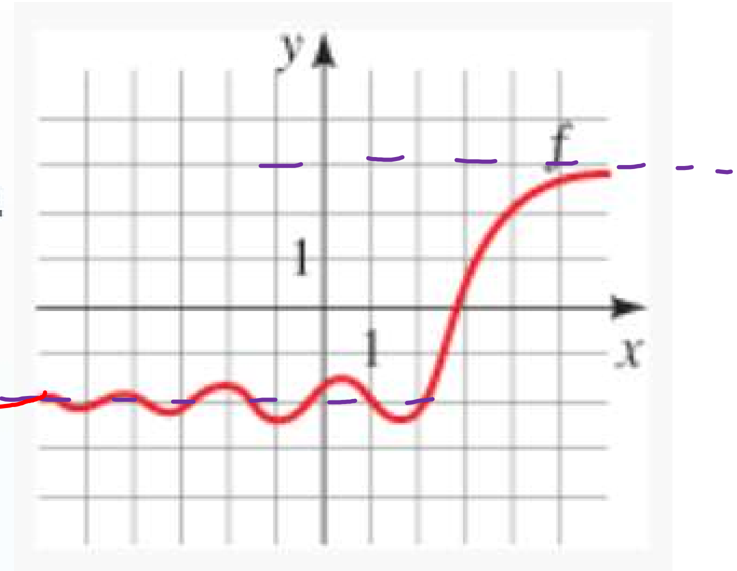
$$\lim_{x\to\infty}\frac{\frac{4\cancel{x^2}}{\cancel{x^2}}}{\frac{5}{x^2}-\frac{2\cancel{x^2}}{\cancel{x^2}}} + 0 + 7$$

$$\frac{4}{0-2}+7=-2+7=\boxed{5}$$

***Find the limit of each part separately, then add/subtract.**

#4 Limits from a Graph

(a) Use the graph of f to find the following limits.



(i) $\lim_{x\to\infty} f(x) = \boxed{3}$

(ii) $\lim_{x\to-\infty} f(x) = \boxed{-2}$

(b) State the equations of the horizontal asymptotes.

$$\boxed{\begin{array}{l} y = 3 \\ y = -2 \end{array}}$$

9. $\lim\limits_{x \to -\infty} \dfrac{4x^2 + 1}{2 + 3x^2}$

Answer

10. $\lim\limits_{x \to -\infty} \dfrac{x^2 + 2}{x^3 + x + 1}$

11. $\lim\limits_{t \to \infty} \dfrac{8t^3 + t}{(2t - 1)(2t^2 + 1)}$

Answer

12. $\lim\limits_{r \to \infty} \dfrac{4r^3 - r^2}{(r + 1)^3}$

Hint #11-12:
First multiply values in denominator to get rid of parentheses, then solve as in previous problems.

13. $\lim_{x\to\infty} \frac{x^4}{1 - x^2 + x^3}$

Answer

14. $\lim_{t\to\infty} \left(\frac{1}{t} - \frac{2t}{t-1}\right)$

15. $\lim_{x\to-\infty} \left(\frac{x-1}{x+1} + 6\right)$

Answer

#14-15:
Find the limit of each part separately, then add/subtract.

16. $\lim_{x\to-\infty}\left(\frac{3-x}{3+x}-2\right)$

#16: Find the limit of each part separately, then add/subtract.

17. $\lim_{x\to\infty}\cos x$

Answer

#17-18: Solve using a graph.

18. $\lim_{x\to\infty}\sin^2 x$